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Cosmology of Brane-Induced Gravity Models[1]

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We review various aspects of the cosmology of brane-induced gravity models. After recalling some properties of these models, we give the equations governing the cosmological dynamics in a Z_2 symmetric case. We then discuss properties of two particular solutions of interest, a self-accelerating solution that has been proposed to provide an alternative explanation to the observed late time acceleration of the universe, and a self-flattening solution. The latter is also discussed in relation with the van Dam–Veltman–Zakharov discontinuity.

KEY WORDS: cosmology; brane worlds; alternative theories of gravity; cosmic acceleration; massive gravity.

1. INTRODUCTION

The brane-induced gravity models initially proposed in Dvali *et al.* (2000) and further developed in Dvali and Gabadadze (2001) and Dvali *et al.* (2001a,b, 2002), whose cosmology is the subject of this paper, are a particular class of brane-world models. The latter have recently attracted a lot of attention, and can be defined as models where our four-dimensional (4D) universe is considered to be a surface (called a *brane*) embedded into a higher dimensional *bulk* space-time. Brane-world models are inspired by superstring-M theory, and can be regarded as some low-energy effective models of more fundamental underlying theories, but are also of interest on their own. This is particularly true with the brane-induced gravity models, which can provide new phenomenological ideas, but are also a playground to investigate the van Dam–Veltman–Zakharov discontinuity (vDVZ) (Van Dam and Veltman, 1970; Zakharov, 1970). After recalling some properties of those models (this section), we give the equations governing their cosmology when the bulk is symmetric[3] with respect to the brane (Section 2). These equations were first derived in Deffayet (2001). We then review previous works about two particular solutions. The first (discussed in Section 3), a self-inflationary solution

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[3] See Cordero and Vilenkin (2001) and Dick (2001a,b) for discussions of nonsymmetric solutions.

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found in Deffayet (2001), was proposed in Deffayet (2001) and Deffayet *et al.* (2001a) to be used to reproduce the observed late time acceleration of the universe (Perlmutter *et al.*, 1999; Riess *et al.*, 2001; Riess *et al.*, 1998) without the need for a nonzero cosmological constant. It was further compared to SNIa and CMB data in Deffayet *et al.* (2002). The last (discussed in Section 4), introduced in Deffayet *et al.* (2001c), has the property that 4D Minkowski space-time is a late time attractor for a large class of initial conditions on the brane. It was further discussed in Deffayet *et al.* (2001d) in relation with the vDVZ discontinuity.

1.1. Defining Properties of Brane-Induced Gravity Models

We will only consider here the case of a single brane, thought of as our 4D universe, embedded in a five-dimensional (5D) bulk. The first properties of brane-induced gravity models we would like to recall are common with a large class of brane-worlds. We first define the brane embedding into the bulk by the coordinates $X^A(x^\mu)$ of the brane-world volume (parametrized by coordinates x^μ) into the 5D space–time. The bulk metric $\tilde{g}_{AB}$ induces through this embedding $X^A(x^\mu)$ a metric $g_{\mu\nu}$ on the brane (called induced metric) defined by[4]

$$g_{\mu\nu} = \tilde{g}_{AB}\partial_\mu X^A \partial_\nu X^B. \tag{1}$$

In the above equation, and in the following, we put a tilde on quantities (e.g., the 5D metric $\tilde{g}_{AB}$ or the 5D Ricci scalar $\tilde{R}$) to distinguish them from their 4D counterparts depending only on the induced metric (e.g., $g_{\mu\nu}$ or R). The action of the theory contains the usual 5D Einstein–Hilbert action

$$S_{\rm EH} = \frac{M_{(5)}^3}{2} \int d^5X \sqrt{|\tilde{g}|}\tilde{R}, \tag{2}$$

where $M_{(5)}$ denotes the 5D reduced Planck mass. It also contains the action, $S_{\rm m}$, of matter field which are assumed to be localized on the brane. One writes accordingly

$$S_{\rm m} = \int_{\rm brane} d^4x \sqrt{|g|}\mathcal{L}_{\rm m}, \tag{3}$$

where $\mathcal{L}_{\rm m}$ is a matter Lagrangian density. All the terms considered so far are generically considered in brane-world models.

The gravitational action is taken to contain another term, $S_{\rm eh}$, in addition to the 5D Einstein-Hilbert term (2), given by

$$S_{\rm eh} = \frac{M_{\rm Pl}^2}{2} \int_{\rm brane} d^4x \sqrt{|g|}\, R. \tag{4}$$

[4] In the following, we use uppercase Latin letters $A, B, \ldots$ to denote 5D indices, Greek letters from the middle of the alphabet $\mu, \nu, \ldots$ to denote indices parallel to the brane-world volume, the numeral 5 an index transverse to the brane, and Latin letters $i, j, \ldots$ to denote space-like indices parallel to the brane-world volume.

This term is the usual 4D Einstein–Hilbert term computed here on the brane and with the induced metric. In the above equation $M_{\rm Pl}$ is a mass parameter which one allows, in brane-induced gravity models, to be very large in comparison to the other dimensionful parameter of the theory, $M_{(5)}$. A term such as $S_{\rm eh}$ would also arise quite generically in brane-world models. It can be thought of as being induced by quantum corrections involving the coupling between bulk gravity and brane matter (Dvali *et al.*, 2000; Dvali and Gabadadze, 2001) in the spirit of the induced gravity program of Adler (1980a,b, 1982), Capper (1975), and Zee (1982). This is the allowed large hierarchy between $M_{\rm Pl}$ and $M_{(5)}$ that makes all its phenomenological interest.[5] As will be seen below, this term is able to localize gravity on the brane for distances smaller than a critical length, even if the bulk space-time is flat (5D Minkowski), allowing also $M_{\rm Pl}$ to be interpreted as the usual 4D reduced Planck mass. This is in sharp contrast with other types of brane-world models where the recovery of 4D gravity on the brane is achieved assuming the bulk space-time to be either compact [like e.g. in Arkani-Hamed *et al.* (1998, 1999) and Antoniadis *et al.* (1998)] or curved in a very specific way (Randall and Sundrum, 1999a,b). The relaxation of these hypotheses could in turn shed some light on the cosmological constant problem (Deffayet *et al.*, 2001a; Witten, 2000 Dvali *et al.* (2000)), in addition to the various other virtues of the model, some of which will be discussed below.

Up to a suitable Gibbons–Hawking term, the action of the theory we are considering here is thus given by the sum of Eqs. (2)–(4), and the equation of motion is given by

$$\tilde{G}_{AB} \equiv \tilde{R}_{AB} - \frac{1}{2}\tilde{R}\tilde{g}_{AB} = \frac{1}{M_{(5)}^3}\tilde{T}_{AB}. \tag{5}$$

In the above equation the effective energy–momentum tensor, $\tilde{T}_{AB}$, is given by the sum of the energy–momentum tensor $T_{\mu\nu}$ of the brane and a term proportional to the 4D Einstein tensor $G_{\mu\nu}$ coming from the induced gravity term (4). Namely, choosing a Gaussian normal coordinate system with respect to the brane, where the brane sits at $y = 0$, and $X^A(x^\mu) = \delta^A_\mu x^\mu$, the only nonvanishing components of $\tilde{T}_{AB}$ are

$$\tilde{T}_{\mu\nu} = \delta(y)\left(T_{\mu\nu} - M_{\rm Pl}^2 G_{\mu\nu}\right). \tag{6}$$

1.2. Perturbative Properties

We review here some results obtained in Dvali *et al.* (2000) by a perturbative expansion over a flat (5D Minkowski) background. In this way one can compute the

[5] This hierarchy can be generated e.g. assuming that the standard model U.V. cutoff is much higher than the quantum gravity scale (Dvali *et al.*, 2002).

gravitational potential between static point-like sources on the brane. We first drop the tensorial structure of the graviton propagator and only discuss the dependence with distance of the potential. This is the same as in a scalar field theory where the scalar field action would be given by the sum of a bulk term (5D) and a brane localized (4D) term as in

$$\frac{M_{(5)}^3}{2}\int d^5X\,\partial_A\Phi\partial^A\Phi + \frac{M_{\rm Pl}^2}{2}\int_{\rm brane} d^4x\,\partial_\mu\Phi\partial^\mu\Phi \tag{7}$$

The results of Dvali *et al.* (2000) read for the potential of a unit mass,

$$V(r) = -\frac{1}{4\pi^2 M_{\rm Pl}^2}\,\frac{1}{r}\,\left\{\sin\left(\frac{r}{r_{\rm c}}\right)\mathrm{Ci}\left(\frac{r}{r_{\rm c}}\right) + \frac{1}{2}\,\cos\left(\frac{r}{r_{\rm c}}\right)\left[\pi - 2\mathrm{Si}\left(\frac{r}{r_{\rm c}}\right)\right]\right\}, \tag{8}$$

where $\mathrm{Ci}(z) \equiv \gamma + \ln(z) + \int_0^z(\cos(t)-1)\,dt/t$, $\mathrm{Si}(z) \equiv \int_0^z \sin(t)\,dt/t$, $\gamma \simeq 0.577$ is the Euler–Mascheroni constant. The distance scale $r_{\rm c}$ is given by

$$r_{\rm c} \equiv \frac{M_{\rm Pl}^2}{2M_{(5)}^3}. \tag{9}$$

At short distances when $r \ll r_{\rm c}$, the first leading contributions to $V(r)$ are

$$V(r) \simeq -\frac{1}{4\pi^2 M_{\rm Pl}^2}\frac{1}{r}\left\{\frac{\pi}{2} + \left[-1 + \gamma + \ln\left(\frac{r}{r_{\rm c}}\right)\right]\left(\frac{r}{r_{\rm c}}\right) + \mathcal{O}(r^2)\right\}, \tag{10}$$

so that at short distances, the potential has the 4D Newtonian $1/r$ scaling. This is subsequently modified by the logarithmic *repulsion* term in Eq. (10). The large distance behavior, on the other hand, is given for $r \gg r_{\rm c}$ by

$$V(r) \simeq -\frac{1}{4\pi^2 M_{\rm Pl}^2}\frac{1}{r}\left\{\frac{r_{\rm c}}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)\right\}. \tag{11}$$

Thus, at large distance, the potential scales as $1/r^2$ similarly with laws of 5D theory. The gravitational potential thus exhibits a short distance 4D behavior and a large distance 5D behavior in contrast to the standard brane-world picture where gravity is modified at short distance only.[6]

The mode analysis of the gravitational (or scalar) fluctuations leads to a convenient interpretation of this potential in terms of Kaluza–Klein (KK) modes (Dvali *et al.*, 2001b). Namely there is a continuum of 4D massive KK states, Φ_m, the wave functions of which are suppressed on the brane by

$$|\Phi_m(y=0)|^2 = \frac{1}{4 + m^2 r_{\rm c}^2}, \tag{12}$$

[6] In this particular scenario, one also expects short distance modifications, e.g. when quantum gravity effects become relevant. This will however happen only at distances of order $M_{(5)}^{-1}$ which can be chosen to be much smaller than $r_{\rm c}$.

so that the gravitational potential on the brane is also given by

$$V(r) \propto \frac{1}{M_{(5)}^3} \int_0^\infty \frac{dm}{4 + m^2 r_c^2} \frac{e^{-mr}}{r}, \tag{13}$$

and gravity is mediated by massive modes although its short distance behavior mimics a zero-mode mediation.

Before discussing the relevant numerical parameters, let us turn to the tensorial structure of the graviton propagator. Following again Dvali *et al.* (2000) we consider 5D metric fluctuations $\tilde{h}_{AB}$ over a 5D Minkowski background $\tilde{\eta}_{AB}$. Choosing the harmonic gauge in the bulk

$$\partial^A \tilde{h}_{AB} = \frac{1}{2} \partial_B \tilde{h}^C_C, \tag{14}$$

and setting (consistently with the equations of motion)

$$\tilde{h}_{\mu 5} = 0, \tag{15}$$

the surviving components of $\tilde{h}_{AB}$ are $\tilde{h}_{\mu\nu}$ and $\tilde{h}_{55}$. After some further simplifications, one is led to the equation

$$\left(\frac{M_{(5)}^3}{2} \partial_A \partial^A + \frac{M_{\rm Pl}^2}{2} \delta(y) \partial_\mu \partial^\mu \right) \tilde{h}_{\mu\nu}(x, y) = \left\{ T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T^\alpha_\alpha \right\} \delta(y) + \frac{M_{\rm Pl}^2}{2} \delta(y) \partial_\mu \partial_\nu \tilde{h}^5_5, \tag{16}$$

which encodes the tensor structure of the graviton propagator on the brane.[7] This structure is one of a massive 4D graviton or, equivalently, that of a massless 5D graviton. It is given by

$$D^{\mu\nu\alpha\beta} = \frac{1}{2} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} \eta^{\mu\beta} \eta^{\nu\alpha} - \frac{1}{3} \eta^{\mu\nu} \eta^{\alpha\beta} + \mathcal{O}(p), \tag{17}$$

where we have neglected momentum-dependent terms, and $\eta_{\mu\nu}$ is a 4D Minkowski metric. Namely, the amplitude between two brane sources with conserved energy momentum tensors $T_{\mu\nu}$ and $T^\star_{\mu\nu}$ is thus given in the Fourier Euclidean space by

$$\hat{h}_{\mu\nu}(p, y = 0) \hat{T}^{\mu\nu}(p) \propto \frac{\hat{T}^{\mu\nu} \hat{T}^\star_{\mu\nu} - \frac{1}{3} \hat{T}^\mu_\mu \hat{T}^{\star\nu}_\nu}{M_{\rm Pl}^2 p^2 + 2 M_{(5)}^3 p}, \tag{18}$$

where the accented quantities are Fourier components, and p is the Euclidean norm of the 4-momentum. This means that, in the small distance, $r \ll r_c$, regime, if one wants to have the usual 4D expression for the force between two static point masses on the brane, one needs to rescale the Newton constant, $G_N = M_{\rm Pl}^{-2}/8\pi$,

[7] One has also $\partial_\mu \partial^\mu \tilde{h}^\nu_\nu = \partial_\mu \partial^\mu \tilde{h}^5_5$.

defined as usual from the action (4), by a factor $3/4$. This rescaling is independent of r_c and thus persists in the limit of $r_c \to \infty$, leading to the celebrated vDVZ discontinuity (Van Dam and Veltman, 1970; Zakharov, 1970). This discontinuity, if real in the full nonperturbative solution, would definitely be enough to rule out the model, since it leads, e.g., to a different prediction for light bending than the one of General Relativity.[8] However, as will be discussed below, the exact cosmological solutions found in the model can indeed give a strong indication in favor of the claim once made by Vainshtein (1972) that this discontinuity disappears in the full (nonperturbative) exact solution. This has also been confirmed by more recent works by Lue (2001) and Gruzinov (2001), and we will come back to this question in Section 4.

Eventually, we would like to discuss the numerical values of some relevant quantities. The only dimensionful parameter of the theory (except $M_{\rm Pl}$ which is fixed by the small distance regime to its usual value) is $M_{(5)}$ or equivalently r_c. Apart from cosmological bounds, that will be discussed below, the most stringent bound on r_c comes from looking at the first correction to Schwarzschild solution (Gruzinov, 2001) in solar system observations (Talmadge *et al.*, 1988). One finds in this case $r_c \geq 100$ Mpc in agreement with bounds on large distance modification of gravity (Goldhaber and Nieto, 1974; Groom *et al.*, 2000). This leads in turn to an estimation for $M_{(5)} \leq 100$ MeV. This low quantum gravity scale leads however to no conflict with experiments (Dvali *et al.*, 2001b), and the brane-induced gravity models have indeed been proposed as providing a framework to realize very low-scale quantum gravity theories (Dvali *et al.*, 2002). We refer the interested reader to Dvali *et al.* (2001b, 2002) and Gia Dvali's contribution for more details.

2. COSMOLOGICAL DYNAMICS

2.1. Friedmann's Equations

We now briefly derive the Friedmann's equations for the model considered here recalling results obtained in Deffayet (2001). We start with an ansatz for the metric of the form

$$ds^2 = -n^2(\tau, y)\, d\tau^2 + a^2(\tau, y)\gamma_{ij}\, dx^i\, dx^j + dy^2, \tag{19}$$

where γ_{ij} is a maximally symmetric Euclidean three-dimensional metric ($k = -1, 0, 1$ will parametrize the spatial curvature). The brane matter energy–momentum tensor is taken accordingly with the following symmetry

$$T^A{}_B = \delta(y)\, \mathrm{diag}\,(-\rho,\, p,\, p,\, p,\, 0)\,, \tag{20}$$

[8] See e.g. Giannakis and Ren (2001) where the linearized Schwarzschild solution in the model Dvali *et al.*, 2000 has been worked out.

where ρ and p are the energy density and pressure of the matter cosmic fluid. Considering here only a bulk with a vanishing cosmological constant, the Einstein's equations in the bulk can be solved by the first integral

$$\left(a'a\right)^2 - \frac{(\dot{a}a)^2}{n^2} - ka^2 + \mathcal{C} = 0, \tag{21}$$

where $\mathcal{C}$ is a constant of integration, a prime denotes a derivation with respect to y, and a dot a derivation with respect to τ. It is also possible to obtain the explicit form of the bulk metric, but this is not discussed here (see Deffayet (2001)). The brane is then taken into account by using Israel–Darmois junction conditions (Israel, 1966; Darmois, 1927), which relate the jump across the brane of the brane extrinsic curvature to the delta functions sources on the right-hand side of Einstein's equations (5). They read here

$$\frac{[a']}{a_b} = -\frac{\rho}{3M_{(5)}^3} + \frac{M_{\rm PI}^2}{M_{(5)}^3 n_b^2}\left\{\frac{\dot{a}_b^2}{a_b^2} + k\frac{n_b^2}{a_b^2}\right\}, \tag{22}$$

$$\frac{[n']}{n_b} = \frac{3p + 2\rho}{3M_{(5)}^3} + \frac{M_{\rm PI}^2}{M_{(5)}^3 n_b^2}\left\{-\frac{\dot{a}_b^2}{a_b^2} - 2\frac{\dot{a}_b\dot{n}_b}{a_b n_b} + 2\frac{\ddot{a}_b}{a_b} - k\frac{n_b^2}{a_b^2}\right\}, \tag{23}$$

where the subscript b for a, b, n means that these functions are taken in $y = 0$, and $[Q] = Q(0^+) - Q(0^-)$ denotes the jump of the function Q across $y = 0$. We can compare Eqs. (22) and (23) with the similar equations obtained when discarding the term (4) in the action [see e.g. Binetruy *et al.* (2000b)]. The latter are recovered by letting $M_{\rm PI}$ go to zero. This also shows explicitly that for a given induced metric parametrized by a_b, n_b,[9] and k, the intrinsic curvature term (4) acts as a "cosmic fluid"[10] of density $\rho_{\rm curv}$ and pressure $p_{\rm curv}$ given by

$$\rho_{\rm curv} = -\frac{3M_{\rm PI}^2}{n_b^2}\left\{\frac{\dot{a}_b^2}{a_b^2} + k\frac{n_b^2}{a_b^2}\right\}, \tag{24}$$

$$p_{\rm curv} = \frac{M_{\rm PI}^2}{n_b^2}\left\{\frac{\dot{a}_b^2}{a_b^2} - 2\frac{\dot{a}_b\dot{n}_b}{a_b n_b} + 2\frac{\ddot{a}_b}{a_b} + k\frac{n_b^2}{a_b^2}\right\}. \tag{25}$$

One notes that the energy density of this "fluid" is always negative whenever $k = 0$ or $k = 1$. Assuming the symmetry[11] $y \leftrightarrow -y$, the junction condition (22) can be used to compute a' on the two sides of the brane. We have in this case $[a'] = 2a'(0^+)$. By continuity when $y \to 0$, Eq. (21) yields the generalized (first)

[9] n_b can also be eliminated by a suitable change of time coordinate.

[10] The 4D Bianchi indentities $\nabla^\mu G_{\mu\nu} = 0$ ensure that the energy–momentum of this "fluid" is conserved.

[11] This choice matches the symmetry of the propagator computed in Dvali *et al.* (2000) leading to Eq. (8).

Friedmann's equation:

$$\epsilon\sqrt{H^2 - \frac{\mathcal{C}}{a_b^4} + \frac{k}{a_b^2}} = \frac{M_{\rm Pl}^2}{2M_{(5)}^3}\left(H^2 + \frac{k}{a_b^2}\right) - \frac{\rho}{6M_{(5)}^3}, \tag{26}$$

where the Hubble parameter H is defined here by

$$H = \frac{\dot{a}_b}{a_b n_b}, \tag{27}$$

and $\epsilon = \pm 1$ is the sign of $[a']$ [see Eq. (22)]. The two different possible choice of ϵ correspond to two different embeddings of the brane into the bulk space-time [see Deffayet (2001) and e.g. Bowcock *et al.* (2002), Cvetič *et al.* (1993), and Gibbons (1993)] . If we plug into the (0, 5) component of the Einstein's equations the jump conditions (22) and (23) we obtain, as when no brane intrinsic curvature (4) is included, the conservation equation

$$\dot{\rho} + 3(p + \rho)\frac{\dot{a}_b}{a_b} = 0. \tag{28}$$

Equations (26) and (28), together with the brane matter equation of state, are then sufficient to derive the cosmological evolution of the brane metric. We note eventually that a nonzero $\mathcal{C}$ means that the Weyl's tensor of the bulk does not vanish (Mukohyama *et al.*, 2000; Shiromizu *et al.*, 2000). Since we are mainly interested here in cases where the bulk is Minkowskian, we will set $\mathcal{C}$ to zero in the rest of this work.

2.2. Early Time Cosmology

The Friedmann's equation (26) shows that usual 4D cosmology is recovered whenever the term in the left-hand side of the equation is subdominant with respect to the first term in the right-hand side, or namely when

$$\sqrt{H^2 + \frac{k}{a_b^2}} \gg 2\frac{M_{(5)}^3}{M_{\rm Pl}^2}. \tag{29}$$

This is reexpressed in terms of the Hubble radius, H^{-1}, and $r_{\rm c}$, by (neglecting the spatial curvature)

$$H^{-1} \ll r_{\rm c}. \tag{30}$$

In this regime Eq. (26) reduces at leading order to the standard 4D Friedmann's equation

$$H^2 + \frac{k}{a_b^2} = \frac{8\pi G_N \rho}{3}. \tag{31}$$

This confirms the perturbative calculation done in Dvali *et al.* (2000) (namely that the small distance–large curvature behavior of gravity is standard), with however the following important difference: the Newton's constant entering into Eq. (31) is G_N (and not $4/3G_N$ for, e.g., nonrelativistic matter, as could have been expected from the perturbative calculation and a Newtonian cosmology argument, see Deffayet *et al.* (2001d) for more details), so that there is no appearance of the vDVZ discontinuity in the exact cosmological solutions. We will come back to this question later.

2.3. Late Time Cosmology

To investigate the late time cosmological evolution, let us rewrite the Friedmann's equation (26) as

$$H^2 + \frac{k}{a_b^2} = \left(\sqrt{\frac{\rho}{3M_{\rm PI}^2} + \frac{1}{4r_c^2}} + \epsilon \frac{1}{2r_c} \right)^2. \tag{32}$$

Equation (29), which gives the domain of validity of the standard-like early cosmology, can be rewritten as

$$\rho \gg \rho_c, \tag{33}$$

where ρ_c is defined by

$$\rho_c = \frac{3M_{\rm PI}^2}{4r_c^2}. \tag{34}$$

Let us now assume that the 4D brane universe is endowed with a matter content such that its energy density decreases with cosmological time and does not asymptote any nonzero value (e.g. matter or radiation[12]). Starting from an early phase where Eq. (33) holds, one then reaches a regime where ρ gets much lower than ρ_c (this happens equivalently when the Hubble radius H^{-1} gets much larger than r_c). One sees then that there are two different asymptotic dynamics depending on the sign of ϵ.

Let us first look at the case where ϵ is equal to -1. In this case, when the matter energy density decreases, one is led to a regime where the Friedmann's equation (26) is given at leading order by

$$H^2 + \frac{k}{a_b^2} = \frac{\rho^2}{36M_{(5)}^6}. \tag{35}$$

This is the relation one would have obtained neglecting the term (4) in the action of the theory, and is typical of brane cosmology in a bulk with no cosmological constant (Binétruy *et al.*, 2000a,b).

[12] This will be true for any kind of matter having the same property in standard 4D General Relativity since the conservation equation Eq. (28) is the usual one.

On the other hand, when ϵ is equal to 1, the left-hand side of Eq. (32) is bounded from below by $1/r_c^2$, and the universe evolves to a de Sitter phase, even if the cosmological constant vanishes. The model considered here has indeed de Sitter space-time as a vacuum solution, as can be understood recalling that the brane-induced gravitational kinetic term (4) acts as a brane-localized source [Eqs. (24) and (25)] to the 5D Einstein's equations, so that one has a nontrivial vacuum solution even in the absence of "matter" sources. This is in strong analogy with models of inflation sourced by higher derivative terms (Mijic *et al.*, 1986; Starobinsky, 1980). The above discussion shows that in order for the model considered here to be compatible with the known successes of cosmology, one needs the crossover radius r_c to be large enough. A conservative bound is that r_c should be of the order of, or larger than, today's Hubble radius. This is also compatible with other bounds quoted at the end of Section 1.2. With such a choice of parameters, one does not spoil successes of the hot Big Bang such as nucleosynthesis, and the evolution of the universe is standard, with deviation (if any) only occurring at very recent times.

We would like to now briefly review two interesting solutions, each pertaining to one of the two branches of solutions mentioned above, and discuss some of their possible virtues.

3. THE LATE TIME ACCELERATING SOLUTION

As mentioned above, the late time dynamics of the $\epsilon = 1$ branch of solutions asymptotes a de Sitter phase when the energy density of the universe is decreasing to a sufficiently small value. This is the basis of the proposition made in Deffayet (2001) and Deffayet *et al.* (2001a) to use this branch to explain the observed late time acceleration of the universe (Perlmutter *et al.*, 1999; Riess *et al.*, 1998, 2001) without the need for a nonvanishing cosmological constant. This idea was further explored in Deffayet *et al.* (2002), fitting for the cosmological parameters using SNIa and CMB data. In the following subsections we first briefly compare the outcome of this proposal to standard cosmology with various forms of dark energy (Section (3.1)), and then present the results obtained in Deffayet *et al.* (2002) for the parameters estimations (Section (3.2)).

3.1. Comparison With Standard Cosmology

Using the conservation equation (28), Eq. (32) gives the Hubble parameter H as a function of redshift z by

$$H^2(z) = H_0^2 \left\{ \Omega_k(1+z)^2 + \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \sum_\alpha \Omega_\alpha (1+z)^{3(1+w_\alpha)}} \right)^2 \right\}, \tag{36}$$

where we have assumed that ρ is given by the sum of the energy densities ρ_α of different components (labeled by α) with constant equation of state parameters w_α. The Ω's for matter and curvature are defined in the usual way by

$$\Omega_\alpha \equiv \frac{\rho_\alpha^0}{3{M_{\rm Pl}}^2 H_0^2 a_0^{3(1+w_\alpha)}}, \tag{37}$$

$$\Omega_k \equiv \frac{-k}{H_0^2 a_0^2}, \tag{38}$$

$$\tag{39}$$

whereas Ω_{r_c} is given by

$$\Omega_{r_c} \equiv \frac{1}{4r_c^2 H_0^2}. \tag{40}$$

The normalization condition for the Ωs,

$$\Omega_k + \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \sum_\alpha \Omega_\alpha}\right)^2 = 1, \tag{41}$$

differs from the usual relation $\Omega_k + \sum_\alpha \Omega_\alpha = 1$. In the following we will then only consider a universe with a zero cosmological constant, and usual (dark, baryonic, etc.) matter content. It is then apparent from Eq. (36) that Ω_{r_c} acts in a way similar to a cosmological constant in standard Friedmann's equations. To be more accurate, the above described cosmology is exactly reproduced by standard cosmology with a dark energy component with a z-dependent equation of state parameter $w_X^{\rm eff}(z)$, which for a universe containing only nonrelativistic matter, is given by [see Deffayet *et al.* (2001a)]

$$w_X^{\rm eff}(z) = \frac{1}{\left(\sqrt{\frac{4\Omega_{r_c}}{\Omega_M(1+z)^3}} + 4\right)\left(\sqrt{\frac{\Omega_{r_c}}{\Omega_M(1+z)^3}} + \sqrt{\frac{\Omega_{r_c}}{\Omega_M(1+z)^3} + 1}\right)} - 1. \tag{42}$$

At large redshift $w_X^{\rm eff}$ tends toward $-\frac{1}{2}$, reflecting the fact that the dominant term in Eq. (36), after matter and curvature terms, redshifts as $(1+z)^{3/2}$ at large z. At low z, however, $w_X^{\rm eff}$ decreases toward an (Ω_k, Ω_M)-dependent asymptotic value. For a flat universe, the latter is simply given by[13] $-1/(1+\Omega_M)$. Figure 1 shows the different possibilities for the expansion as a function of Ω_M and Ω_{r_c}.

3.2. Fits to SNIa and CMB Data

In order to compare the outcome of the cosmology (36) and the SNIa observations, one uses the standard expression for the luminosity distances $d_{\rm L}$ (since

[13] For example, for $\Omega_M = 0.3$ and $k = 0$, $w_X^{\rm eff}$ at low z tends toward -0.77.

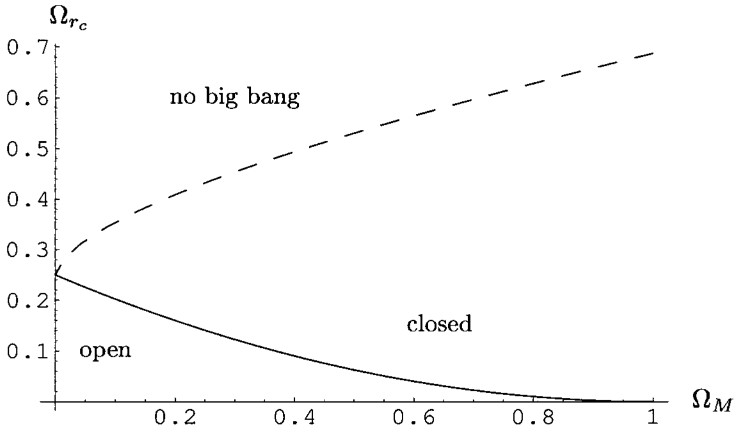

Fig. 1. Different possibilities for the expansion as a function of Ω_M and Ω_{r_c}. The solid line denotes a flat universe ($k = 0$), with Ω_{r_c} obtained through Eq (41). The universes above the solid line are closed ($k = 1$), the universes below are open ($k = -1$). The universes above the dashed line avoid the big bang singularity by bouncing in the past.

this is only dependent of the form of the metric on the brane, which is the usual FLRW form) as a function of the redshift z given by

$$d_{\mathrm{L}} = H_0^{-1}(1+z)\frac{S_k\left(\sqrt{|\Omega_k|}d_{\mathrm{C}}(z)\right)}{\sqrt{|\Omega_k|}}, \tag{43}$$

with $d_{\mathrm{C}}(z)$ defined by

$$d_{\mathrm{C}}(z) = \int_0^z H_0 \frac{dy}{H(y)}, \tag{44}$$

$H(z)$ is given by Eq. (36), and S_k reads

$$S_k(r) = \begin{cases} \sin r & (k=1) \\ \sinh r & (k=-1) \\ r & (k=0) \end{cases}. \tag{45}$$

Figure 2 shows the luminosity distances for various values of the parameters of standard cosmology compared to the outcome of the cosmology (36).

A fit to the supernovae data set from the SCP (Perlmutter, 1999), with the luminosity distance calculated using Eq. (36), yields the contours reproduced in Fig. 3. For a flat universe, the results of the χ^2 minimization gives (one sigma levels)

$$\Omega_M = 0.18^{+0.07}_{-0.06} \quad \text{or} \quad \Omega_{r_c} = 0.17^{+0.03}_{-0.02}, \tag{46}$$

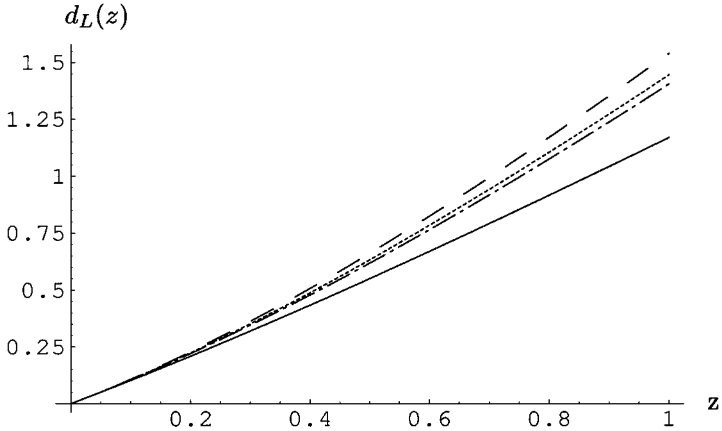

Fig. 2. Luminosity distance as a function of redshift for ordinary cosmology with $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$, $k = 0$ (dashed line), $\Omega_\Lambda = 0$, $\Omega_M = 1$, $k = 0$ (solid line), and dark energy with $\Omega_X = 0.7$, $w_X = -0.6$, $\Omega_M = 0.3$, $k = 0$ (dotted-dashed line) and in our model (dotted line) with $\Omega_M = 0.3$ and a flat universe (for which one gets from Eq. (41) $\Omega_{r_c} = 0.12$ and $r_c = 1.4 H_0^{-1}$).

with $\chi^2 = 57.96$, for 52 (54 SNe −2 parameters) degrees of freedom.[14] This leads to an estimate r_c in terms of the Hubble radius given by

$$r_c = 1.21^{+0.09}_{-0.09} H_0^{-1}. \tag{47}$$

The degeneracy appearing in the (Ω_M, Ω_{r_c}) plane can be lifted by comparison with CMB data. For that purpose a modified version of CMBFAST (Seljak and Zaldarriaga, 1996) was used in Deffayet *et al.* (2002) replacing the first Friedmann's equation by Eq. (32). The equations for the growth of cosmological perturbations were kept the same as in usual cosmology (except for the background evolution). This is justified for the small-scale perturbations and for processes happening early enough in the history of the universe, as is discussed in more detail in Deffayet *et al.* (2002). On the other hand, one can expect deviations from the standard picture at large scale (and late time) where (and when) the effect of the extra dimension began to be felt. This concerns scales of order of today's Hubble radius and processes happening in the late history of the universe. A more careful exploration of the approximation made in Deffayet *et al.* (2002) has still to be carried out Deffayet *et al.* (in preparation). This could potentially lead to a way to discriminate between standard cosmology and the model considered here by, e.g., data on the large-scale structures.

[14] These numerical results are in agreement with the fit done in Avelino and Martins (2002); we however disagree with the conclusions of that work [see Deffayet *et al.* (2002) and Deffayet *et al.* (2001b) for a discussion of this paper].

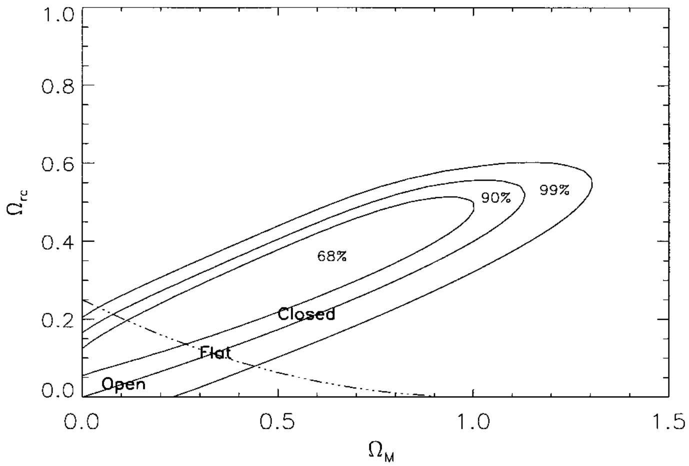

Fig. 3. Confidence regions (68.3%, 90%, and 99%) for (Ω_M, Ω_{rc}) in gravitational leakage scenario, assuming no prior knowledge of α and $\mathcal{M}$.

In Deffayet *et al.* (2002), a six-dimensional parameter space, $\theta = (\Omega_k, \Omega_{r_c}, \omega_d, \omega_b, n, A)$, where $\omega_d = \Omega_{cdm}h^2$, $\omega_b = \Omega_b h^2$, and A and n are the amplitude and slope of the primordial spectrum of perturbations was explored using a Markov chain method. The details of the procedure can be found in the original reference (Deffayet *et al.*, 2002). Figure 4 shows the results of the analysis in the $\Omega_M - \Omega_{r_c}$ plane. The shaded region was drawn to contain approximately 95% of the models in the chain; the line marks the location of spatially flat models. The constraint on Ω_{r_c} is coming mainly from the position of the acoustic peaks and so there is a natural degeneracy in the Ω_m–Ω_{r_c} plane which is apparent in the plot.

As expected the CMB data prefers spatially flat models. Thus it is natural to further restrict the analysis to flat universes, which was done by considering only samples in the chain with negligible curvature. The probability distribution for Ω_M under this assumption is shown in Fig. 5.

The fits done in Deffayet *et al.* (2002) show that the model of accelerated universe proposed in Deffayet (2001) and Deffayet *et al.* (2001a) is in current agreement with SNIa and CMB data. The degeneracies in parameters estimations using one data set (e.g. CMB) can be partially lifted using the other (e.g. SNIa) as in standard cosmology. The supernovae data prefer a slightly lower value of Ω_M ($\Omega_M = 0.18^{+0.07}_{-0.06}$) than the CMB for a flat universe; however, a concordance model with $(\Omega_k, \Omega_{r_c}, \omega_d, \omega_b, n, A) = (0, 0.1225, 0.1, 0.02, 0.96, 0.57)$, which has $\Omega_M = 0.3$ (and $\chi^2 \approx 140$ for the full data set (135 data points)), provides a good fit to both sets, all the more as systematic errors have not been included in the parameter estimations. For this model the crossover distance between 4D and 5D gravity is given by $r_c \sim 1.4 H_0^{-1}$.

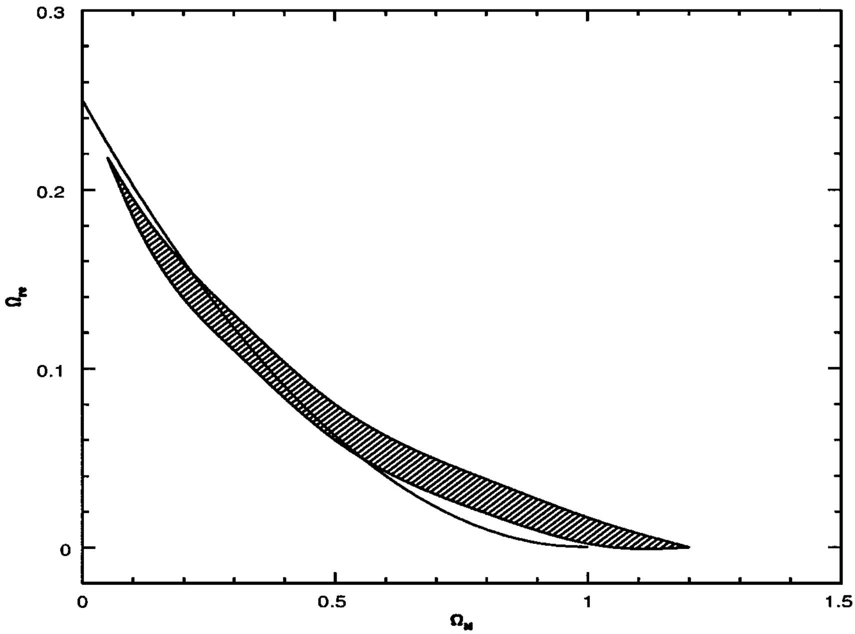

Fig. 4. Allowed region in the $\Omega_M - \Omega_{r_c}$ plane (shaded). The line shows the location of spatially flat models. The shaded region was drawn to contain approximately 95% of the models in our chain.

We also underline here that the model under consideration is very predictive in the sense that future observations have the potential to rule it out. In contrast to quintessence models, this model has the same number of free parameters as the usual LCDM model. With the advent of new precision cosmological measurements such as new SNIa observations, CMB measurements, ongoing galaxy surveys such as Sloan and 2dF, weak lensing surveys, etc, it should be possible to test the model very accurately. Another possible way to discriminate between this model and standard cosmology relies on a better understanding of cosmological perturbations, as has been mentioned above.

4. THE SELF-FLATTENING SOLUTION AND vDVZ DISCONTINUITY

We now turn to discuss some aspects of the $\epsilon = -1$ branch of solution of Eq. (26). We first start by describing some properties of one of these solutions given in Deffayet *et al.* (2001c) and further discussed in Deffayet *et al.* (2001d) in relation with the vDVZ discontinuity. In this particular solution, the brane is endowed with a negative cosmological constant Λ, in addition to "ordinary" matter. All what is needed to get the cosmology is then to replace ρ by $\rho - |\Lambda|$ and p by $p + |\Lambda|$ in

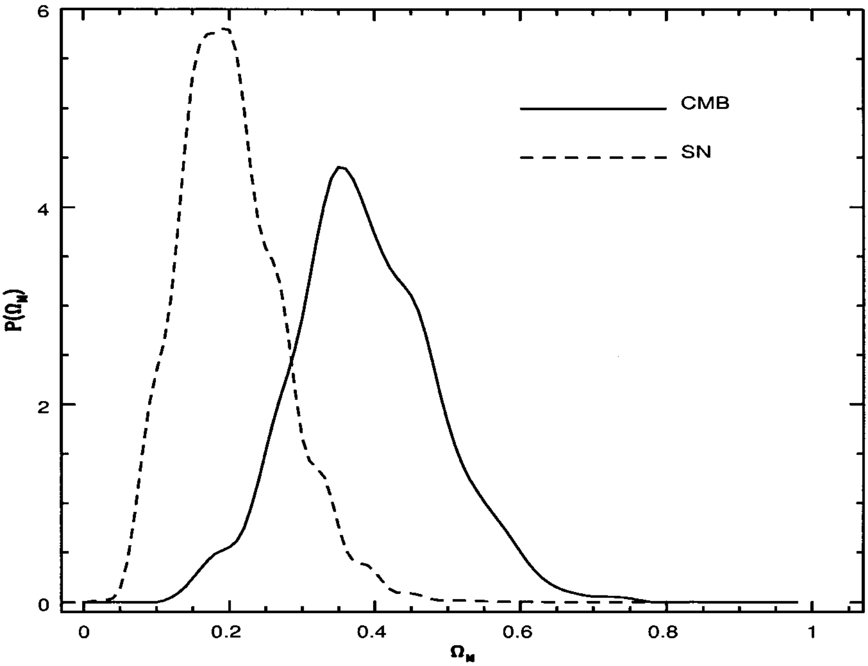

Fig. 5. Marginal distributions for Ω_M under the assumption that the universe is spatially flat. The solid line shows the results from CMB and the dashed line from SN.

Eq. (26) and (28), where ρ and p now represent the energy density and pressure of "ordinary" matter respectively. Let us then consider matter with equation of state $p = w\rho$, with a constant w, and $w > -1$, and a spatially flat universe (with $k = 0$). Starting from initial conditions where the total energy density of the universe ρ_{tot}, given by

$$\rho_{\text{tot}} = \rho - |\Lambda|, \tag{48}$$

is positive and such that $\rho \gg |\Lambda|$ together with $\rho \gg \rho_c$, one knows from the previous analysis that the early evolution of the universe follows the standard 4D usual Friedmann's equations. The universe's energy density ρ_{tot} then decreases until it becomes lower than the threshold ρ_c, while still being positive. This signals the entry into the late time asymptotic phase where the Friedmann's Eq. (26) is given at leading order by the pure 5D brane cosmology Eq. (35). One can then show that ρ_{tot} asymptotes to zero in infinite cosmological time, this being due to the particular form of Eq. (35). This is in striking contrast with ordinary cosmology with a negative cosmological constant where the total energy density goes to zero and the universe bounces back in a finite time. The full 5D metric (19) has then the asymptotic form

$$ds^2 = -(1 + |\Lambda y|/2M_{(5)}^3)^2\, d\tau^2 + dx_i\, dx^i + dy^2. \tag{49}$$

This metric in the bulk is of course simply a rewriting of 5D Minkowski flat metric,[15] which in this particular case is easily recognized as being a two-dimensional Rindler space-time a 3D Euclidean space. It has the particularity to violate 4D Lorentz symmetry from $y =$ constant slice to the other. This however can only give rise to observable effect through graviton exchange, since only the graviton propagates in the bulk and sees this violation, and can be made arbitrarily small by conveniently choosing the parameters [see Deffayet *et al.* (2001c) where various aspects of this violation are discussed]. On the other hand, the metric on the brane is 4D Minkowski, while the asymptotic form of the brane energy–momentum tensor is then given by

$$T^A{}_B = \delta(y)\, \mathrm{diag}(0, -|\Lambda|, -|\Lambda|, -|\Lambda|, 0). \tag{50}$$

Such an evolution toward 4D Minkowski space-time on the brane would also hold true for more general forms of matter like, e.g., a scalar field as has been shown in Deffayet *et al.* (2001c). Let us note, however, that if one wants to apply such a mechanism to our universe, one should indeed impose very strong constraints on the parameters of the model. The most stringent constraints come from requiring that the recent history of the universe does not differ dramatically from the standard successful cosmological history, which requires $r_c \geq H_0^{-1}$ and $|\Lambda| \leq 10^{-3}$ eV. This is no better than the usual constraint on the cosmological constant in ordinary gravity and does not give a solution to the cosmological constant problem; however, the above described mechanism provides an interesting way to "prepare" 4D Minkowski out of a very large class of initial conditions.

Another aspect of the $\epsilon = -1$ branch of solution of Eq. (26) is related to the vDVZ discontinuity. As we have mentioned before, all those solutions, in which the brane is endowed with matter which energy density decreases to zero as the universe expands, interpolate between two regimes: an early regime where the cosmology is simply given at leading order by ordinary 4D cosmology (31), and a late time regime where the cosmology is given at leading order by pure 5D brane cosmology (35). In other words those solutions are interpolating between exact solutions of two theories: Theory I, defined by the sum of actions (3) and (4), is just ordinary 4D gravity, and Theory II, defined by the sum of the actions (2) and (3) (and the Gibbons–Hawking term), is a 5D brane-world theory. This interpolation can be obtained by tuning continuously in the cosmological solutions given here of the full theory [Theory III defined by the sum of the three terms (2),

[15] As is Eq. (19) for the solutions discussed in this paper, since we have taken the bulk cosmological constant and the bulk Weyl's tensor to vanish.

(3), and (4)] the parameter r_c between 0 (Theory II) and $+\infty$ (Theory I). This means in particular that the solutions considered here have a continuous limit toward those of Theory I, in contrast from what would have been expected from the perturbative analysis recalled in Section 1.2. From this analysis, indeed, one would have expected that one would not recover solutions of Theory I from the limit $r_c \to \infty$, simply because the tensorial structure of the graviton propagator of Theory III is the one of 5D gravity, and thus the limit $r_c \to \infty$ should have exhibited the vDVZ discontinuity.

This supports the argument made by Vainshtein (1972) that the vDVZ discontinuity is namely an artifact of the perturbation theory over a flat space-time. We will not recall here the details of this argument. We refer to the original reference, as well as to Deffayet *et al.* (2001d) for more details. Let us only mention that this argument was made in the framework of a Pauli-Fierz theory for massive gravitons, and relied on a careful examination of a Schwarzschild-type solution in this theory. Namely it was shown in Vainshtein (1972) that there was a well-defined perturbative expansion around the ordinary 4D Schwarzschild which was not singular in the limit of the mass of graviton m going to zero, whereas the perturbative expansion over a flat space-time (which exhibits the discontinuity) was shown to be singular in the same limit. However, this reasoning suffers from several drawbacks. First the theory of massive graviton considered in Vainshtein (1972) is not unambiguously defined. Second it was not verified that it was possible to match the right asymptotic behavior at large radial distance,[16] r from the well-behaved (as m goes to zero) perturbative expansion. The latter was shown to be valid only for a restricted range of radial distances given by

$$r_M \ll r \ll r_M^{1/5} m^{-4/5}. \tag{51}$$

where r_M is the (usual) Schwarzschild radius, and m is the graviton mass.

On the other hand, the Theory III considered here is unambiguously defined, and the propagator of the graviton has the same tensorial structure as the one of a massive (or 5D) gravity (with r_c playing the rôle of m^{-1}). Although it has not been shown exactly that the Schwarzschild solution is recovered at small radii [see however Gruzinov (2001) and Lue (2001)], the cosmological solution mentioned above provides an explicit example of interpolation between a small-time 4D tensorial structure and a large-time 5D tensorial structure. The early time tensorial structure has been discussed above. The late time tensorial structure on the other hand is the one of Theory III, obviously the one of 5D gravity. This can be seen more explicitly, e.g., looking at the solution described in the first part of this subsection. The late time asymptotic metric (49) can indeed be obtained as an expansion over the flat 5D Minkowski from Eq. (16), where the fact that the $\tilde{g}_{ij}$ components of the metric have no y dependence is directly related to the fact that the source on the

[16] This was further underlined in Boulware and Deser (1972).

right hand side of Eq. (16) vanishes for the asymptotic energy–momentum tensor (50).[17] This would not be the case for a 4D tensorial structure.

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[17] For which $T_{ij} - \frac{1}{3} T \eta_{ij}$ vanishes.

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